King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer Semester 2015-2016
Final Exam, Wednesday August 31, 2016.

Name:

ID#:

Instructions:
1. This exam consists of **nine** pages, including this page and the final reference sheet, containing **six** questions.
2. You have to answer all **six** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **not equally weighed**.
5. This exam is out of **100** points.
6. You have exactly **150** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

Question Number	Maximum # of Points	Earned/Deducted Points
I	**30**	
II	**15**	
III	**15**	
IV	**10**	
V	**20**	
VI	**10**	
Total	**100**	

I. (20 points) Choose the correct answer from the following choices.

1. The inverse of "it rains today only if I drive to work" is
   - (a) if I drive to work, then it rains today.
   - (b) if it rains today, then I drive to work.
   - (c) if I don't drive to work, then it does not rain today.
   - (d) if it does not rain today, then I don't drive to work.
   - (e) none of the above.

2. The following compound proposition $\neg(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow q)$ is a
   - (a) tautology.
   - (b) contradiction.
   - (c) contingency.
   - (d) quantified predicate.
   - (e) none of the above.

3. The truth value of $\forall x((-x)^2 = x^2)$, if the domain consists of $\mathbb{Z}^-$ is
   - (a) Φ.
   - (b) $\mathbb{Z}^-$.
   - (c) $\mathbb{Z}$.
   - (d) $\mathbb{N}$
   - (e) $\mathbb{R}$.

4. Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," and A(x, y) the predicate "x has asked y a question," where the domain consists of all people associated with your school. Then, the statement "Some faculty member has never been asked a question by a student" is represented by
   - (a) $\exists x(F(x) \wedge \forall y(\neg S(y) \vee \neg A(y,x))$
   - (b) $\exists x(F(x) \rightarrow \forall y(\neg S(y) \vee \neg A(y,x))$
   - (c) $\exists x(F(x) \wedge \forall y(S(y) \wedge \neg A(y,x))$
   - (d) $\exists x(F(x) \wedge \forall y(\neg S(y) \vee \neg A(x,y))$
   - (e) $\exists x(F(x) \rightarrow \forall y(\neg S(y) \vee \neg A(x,y))$

5. Given two sets A and B where A is countably infinite and B is uncountably infinite. Then,
   - (a) $A - B$ must be countably infinite.
   - (b) $B - A$ must be uncountably infinite.
   - (c) $A \oplus B$ must be countably infinite.
   - (d) $A \cap B$ must be countably infinite.
   - (e) more than one answer above is true.

6. Suppose $f: \mathbb{N} \rightarrow \mathbb{N}$ has the rule $f(n) = 3n^2 - 1$. Then,
   - (a) f is one to one but not onto.
   - (b) f is onto but not one to one.
   - (c) f is neither one to one nor onto.
   - (d) f is one to one and onto.
   - (e) f is not a function.

7. $\bigcap_{i=1}^{\infty}\left(-1-\frac{1}{i},1+\frac{1}{i}\right)$=
   (a) [–2, 2].
   (b) (–2, 2).
   (c) [-1,1].
   (d) (– 1,1).
   (e) none of the above.

8. The amounts of postage that can be formed using just 4-cent and 11-cent stamps include all positive integer values *n* where
   (a) $n \geq 11$.
   (b) $n \geq 15$.
   (c) $n \geq 19$.
   (d) $n \geq 26$.
   (e) n is none of the above.

N>=32

9. How many positive integers between 1000 and 10000 inclusive are divisible by 5 but not 7
   (a) 2830
   (b) 1544
   (c) 1543
   (d) 1542
   (e) 515

10. A group contains 20 boys and 20 girls. How many ways are there to arrange them in a row if the boys and girls alternate?
   (a) $\binom{41}{20}\binom{40}{20}$
   (b) (20!)(20!)
   (c) 2(20!)(20!)
   (d) (20)(20)
   (e) none of the above

II. (15 points) Basic Counting I. In all questions below, make sure that you clearly justify your answer.

1. (10 points) In how many ways can a photographer at a volleyball match arrange six players, the coach and the referee in a row, if
   (a) the coach must be next to the referee?

   2*7!

   (b) the coach is not next to the referee?

   8!-2*7!

   (c) the referee is positioned somewhere to the left of the coach?

   7!+6!+5!+4!+3!+2!

2. (5 points) How many ways are there to seat five people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

$$\frac{5!}{2*5}$$

III. (15 points) Basic Counting II. In all questions below, make sure that you clearly justify your answer.

1. (3 points) There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?

$$\left\lceil \frac{677}{38} \right\rceil$$

2. (5 points) How many ways are there for 12 sisters and seven brothers to stand in a line so that no two brothers stand next to each other? [*Hint:* First position the sisters and then consider possible positions for their brothers.]

$$12!*C(13,7)$$

3. (7 points)What is the least number of area codes needed to guarantee that the 27 million phones in Saudi Arabia can be assigned distinct 9-digit telephone numbers? (Assume that telephone numbers are of the form *NX-NXX-XXXX*, where the first two digits form the area code, *N* represents a digit from 2 to 9 inclusive, and *X* represents any digit.)

$$\left\lceil \frac{27*10^6}{8*10^6} \right\rceil$$

IV. (10 points) Discrete Probability I. In all questions below, make sure that you clearly justify your answer.

1. (3 points) What is the probability that a five-card poker hand contains cards of five different kinds?

$$\frac{C(13,5) * 4^5}{C(52,5)}$$

2. (3 points) What is the probability that a five-card poker hand contains two pairs (that is, two of each of two different kinds and a fifth card of a third kind)?

$$\frac{C(13,2) * C(4,2) * C(4,2) * 11 * 4}{C(52,5)}$$

3. (4 points) What is the probability that a fair die never comes up an even number when it is rolled eight times?

$$(\frac{1}{2})^8$$

V. (20 points) Discrete Probability II. In all questions below, make sure that you clearly justify your answer.

1. (5 points) What is the probability that Hasan, Waleed, Fouad, and Basem win the first, second, third, and fourth prizes, respectively, in a drawing if 100 people enter a contest and no one can win more than one prize.

$$\frac{1}{100 * 99 * 98 * 97}$$

2. (7 points) What is the probability of 4 preceding 1 and 4 preceding 2 when we randomly select a permutation of {1,2,3,4,5}?

4!+2*3!+2!*2!

3. (8 points) Compute the conditional probability that exactly six heads appear when a fair coin is flipped 10 times, given that the first two flips came up tails?

C(8,6)*.5^8

VI. (10 points) Advanced Counting Techniques. In all questions below, make sure that you clearly justify your answer.

1. (5 points) A string that contains only 0s, 1s, and 2s is called a ternary string.
   (a) Find a recurrence relation for the number of ternary strings of length n that contain two consecutive 0s.

$$a_n = 2*(a_{n-1}+a_{n-2})+3^{n-2}$$

   (b) What are the initial conditions?

$$a_1 = 0$$
$$a_2 = 1$$

2. (5 points) Solve the following recurrence relation with together with the initial conditions given:

$$a_n = 2a_{n-1} - a_{n-2} \; for \; n \geq 2, a_0 = 4, a_1 = 1.$$

$$x^2 - 2x + 1 = 0 \rightarrow (x-1)^2 = 0$$
$$x = 1$$
$$a_n = \alpha n + \beta \text{ , } 4 = \beta, 1 = \alpha + 4 \rightarrow \alpha = -3$$
$$a_n = 4 - 3n$$

Some Useful Formulas

$\mathbb{N} = \{0,1,2,3,\ldots\}$ $\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\}$ $\mathbb{Q}$ = set of rational numbers
$\mathbb{R}$ = set of real numbers

$$\sum_{i=a}^{b} 1 = b-a+1 \quad , \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ where } a \neq 1 \quad , \quad \sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ where } |a|<1,$$

$$\sum_{i=1}^{\infty} i a^{i-1} = \frac{1}{(1-a)^2} \text{ where } |a|<1$$

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$((p) \wedge (q)) \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws